Justifying abstraction: Examples from Integration Theory to 1940 with a focus on F. Riesz and O. Nikodym

Tom Archibald

Making sense of mid-twentieth century mathematical abstraction posed problems for both new and ongoing practitioners. To historicize aspects of processes of generalization and abstraction can be tricky as it is easy to be anachronistic. Hilbert's Grundlagen, for example, indicates the recognition of several possible positions on the nature of axioms, for example as "self-evident", as idealizations of experience, or as rules. Since the axioms interact with definitions, this variation in ideas about axioms is accompanied by different ideas about definitions, ranging from definitions as descriptions to definitions as prescriptions. Description, though, is an equivocal term, since one can be describing an object one thinks of as existing, or as one that we are in a sense designing.

A historical question arises in what ways, and in what terms, do researchers attempt to justify their particular approaches to abstraction and generalization? How do these justifications function? In the first half of the 20th c. they were not merely conventional in my view. In what follows, we discuss some research papers and look at explicit or implicit efforts to explain the value of the approach. Such justifications are so familiar now from textbook and other writing that they are easy to overlook. These various ways of justifying one's approach serve as a kind of guide to how the main models of innovation in twentieth century mathematics became standard.

We look in particular at a set of examples around the "Lebesgue-Nikodym" Decomposition theorem in analysis. This is work in progress.