David Waszek (ÉNS-CNRS.)

Mathematical notations in research contexts: Revisiting Leibniz's analogy between powers and differences

Abstract

It is common nowadays, in at least some parts of the mathematical community, to hear about the importance for research of choosing the ‘right’ or ‘natural’ definitions or concepts. To get a clearer grasp of what is at stake in such remarks, and to understand the long-term history of such methodological attitudes, it is helpful to start from a closely related, but quainter and more tractable theme, namely that of the role of notations.

In this perspective, this talk offers a deep dive into what has often been described as a signal instance of notation-driven discovery: Leibniz and Johann Bernoulli's 1695 discovery of an ‘analogy’ between the powers of a sum and the differentials of a product, i.e., between the formulas for $(x+y)^e$ and $d^e(xy)$ – a discovery which, at first sight, seems closely related to Leibniz's introduction of a peculiar ‘exponential’ notation for differentials (d^2x for ddx, d^3x for dddx, $d^{-1}x$ for $\int x$, etc.). This story is no longer widely known, but in the late 18th and early 19th centuries, it was seen as a paradigmatic example of the influence of notations on the development of mathematics; in 1820, Laplace, discussing the exponential notation for powers in general and this extension of it in particular, went so far as to write that ‘such is the advantage of a well-designed language, that its simplest notations have often become the source of the most profound theories’.

Looking closely at Leibniz's unpublished drafts and correspondence from the 1690s, I will investigate how Leibniz was led to adopting his exponential notation for differentials, and pinpoint the peculiar values that made him glimpse promise in it. I will also examine whether one can really claim that the notation itself, independently of the conceptual motivations Leibniz may have had to introduce it in the first place, played a role in Leibniz's discoveries. As it turns out, it did, but its contribution is subtle and cannot be understood in terms ‘expressive power’: it is not true that the new notation allowed expressing things that could not be expressed without it. The notation did, however, impact not just what symbolic manipulations were easily accessible to practitioners, but also what conjectures were seen as plausible and even what could reasonably be considered as a single, unitary theorem.